BINOMIAL THEOREM

OBJECTIVE PROBLEMS

1. **The coefficients of x^2, x^3 in the expansion of $(3+kx)^9$ are equal. Then k =**

   1) 1 2) 2 3) 3 4) 9/7

2. **If the coefficient of rth, (r+1)th and (r+2)th terms in $(1+x)^{14}$ are in A.P., then the value of r is**

   1) 9 2) 10 3) 11 4) 8

3. **The coefficients in the 5^{th}, 6^{th}, 7^{th} terms in the expansion of $(1+x)^n$ are in A.P. Then n =**

   1) 6 2) 7 3) 8 4) 9

4. 16^{th} **term in the expansion of $(\sqrt{x}-\sqrt{y})^{17}$ is**

   (a) $136xy^7$ (b) $136xy$

   (c) $-136xy^{15/2}$ (d) $-136xy^2$

5. **If the coefficients of T_r, T_{r+1}, T_{r+2} terms of $(1+x)^{14}$ are in A.P., then *r* =**

   (a) 6 (b) 7

   (c) 8 (d) 9

6. **If *p* and *q* be positive, then the coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ will be**

   (a) Equal

   (b) Equal in magnitude but opposite in sign

   (c) Reciprocal to each other

   (d) None of these

7. **If the coefficients of r^{th} term and $(r+4)^{th}$ term are equal in the expansion of $(1+x)^{20}$, then the value of *r* will be**

   (a) 7 (b) 8

   (c) 9 (d) 10

8. **The ratio of the coefficient of terms $x^{n-r}a^r$ and $x^r a^{n-r}$ in the binomial expansion of $(x+a)^n$ will be**

   (a) $x : a$ (b) $n : r$

   (c) $x : n$ (d) None of these

9. **If coefficient of $(2r+3)^{th}$ and $(r-1)^{th}$ terms in the expansion of $(1+x)^{15}$ are equal, then value of *r* is**

(a) 5 (b) 6

(c) 4 (d) 3

10. **In the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$, the coefficient of x^4**

(a) $\frac{405}{256}$ (b) $\frac{504}{259}$

(c) $\frac{450}{263}$ (d) None of these

11. **The coefficient of x^{-7} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$ will be**

(a) $\frac{462a^6}{b^5}$ (b) $\frac{462a^5}{b^6}$

(c) $\frac{-462a^5}{b^6}$ (d) $\frac{-462a^6}{b^5}$

12. **If *A* and *B* are the coefficients of x^n in the expansions of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then**

(a) $A=B$ (b) $A=2B$

(c) $2A=B$ (d) None of these

13. **If the coefficients of p^{th}, $(p+1)^{th}$ and $(p+2)^{th}$ terms in the expansion of $(1+x)^n$ are in A.P., then**

(a) $n^2-2np+4p^2=0$

(b) $n^2-n(4p+1)+4p^2-2=0$

(c) $n^2-n(4p+1)+4p^2=0$

(d) None of these

14. **If the coefficients of 5^{th}, 6^{th} and 7^{th} terms in the expansion of $(1+x)^n$ be in A.P., then *n* =**

(a) 7 only (b) 14 only

(c) 7 or 14 (d) None of these

15. If the coefficient of 4^{th} term in the expansion of $(a+b)^n$ is 56, then n is

(a) 12 (b) 10

(c) 8 (d) 6

16. If the third term in the binomial expansion of $(1+x)^m$ is $-\frac{1}{8}x^2$, then the rational value of m is

(a) 2 (b) $1/2$

(c) 3 (d) 4

17. If the coefficients of p^{th}, $(p+1)^{th}$ and $(p+2)^{th}$ terms in the expansion of $(1+x)^n$ are in A.P., then

(a) $n^2 - 2np + 4p^2 = 0$

(b) $n^2 - n(4p+1) + 4p^2 - 2 = 0$

(c) $n^2 - n(4p+1) + 4p^2 = 0$

(d) None of these

18. If coefficients of $(2r+1)^{th}$ term and $(r+2)^{th}$ term are equal in the expansion of $(1+x)^{43}$, then the value of r will be

(a) 14 (b) 15

(c) 13 (d) 16

19. If x^m occurs in the expansion of $\left(x+\frac{1}{x^2}\right)^{2n}$, then the coefficient of x^m is

(a) $\frac{(2n)!}{(m)!(2n-m)!}$ (b) $\frac{(2n)!3!3!}{(2n-m)!}$

(c) $\frac{(2n)!}{\left(\frac{2n-m}{3}\right)!\left(\frac{4n+m}{3}\right)!}$ (d) None of these

20. If the coefficients of x^7 and x^8 in $\left(2+\frac{x}{3}\right)^n$ are equal, then n is

(a) 56 (b) 55

(c) 45 (d) 15

21. Coefficient of x in the expansion of $\left(x^2+\frac{a}{x}\right)^5$ is

(a) $9a^2$ (b) $10a^3$

(c) $10a^2$ (d) $10a$

22. If the coefficients of second, third and fourth term in the expansion of $(1+x)^{2n}$ are in A.P., then $2n^2-9n+7$ is equal to

(a) – 1 (b) 0

(c) 1 (d) 3/2

23. If in the expansion of $(1+x)^m(1-x)^n$, the coefficient of x and x^2 are 3 and – 6 respectively, then *m* is

(a) 6 (b) 9 (c) 12 (d) 24

24. The coefficients of $(2r+4)^{th}$ and $(r-2)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal. Then r =

1) 6 2) 7 3) 8 4) 4

25. In the expansion of $(1+x)^{20}$, the coefficients of $(r+1)^{th}$ and r^{th} terms are in the ratio 2:1. Then r is

1) 7 2) 6 3) 5 4) 4

26. The 21st and 22nd terms in the expansion of $(1+x)^{41}$ are equal. Then the value of x is

1) 1 2) 2 3) 9/7 4) 1/2

27. The coefficient of x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}+\dots\dots\dots+(1+x)^{30}$ is

(a) ${}^{51}C_5$ (b) ${}^{9}C_5$

(c) ${}^{31}C_6-{}^{21}C_6$ (d) ${}^{30}C_5+{}^{20}C_5$

28. In the expansion of $(1+3x+2x^2)^6$ the coefficient of x^{11} is

(a) 144 (b) 288

(c) 216 (d) 576

29. The middle term in the expansion of $\left(x+\frac{1}{x}\right)^{10}$ is

(a) ${}^{10}C_4\frac{1}{x}$ (b) ${}^{10}C_5$

(c) ${}^{10}C_5x$ (d) ${}^{10}C_7x^4$

30. If the second, third and fourth term in the expansion of $(x+a)^n$ are 240, 720 and 1080 respectively, then the value of *n*

(a) 15 (b) 20

(c) 10 (d) 5

31. The term independent of *x* in $\left(2x-\frac{1}{2x^2}\right)^{12}$ **is**

(a) – 7930 (b) – 495

(c) 495 (d) 7920

32. The term independent of *x* in the expansion $\left(x^2-\frac{1}{3x}\right)^9$ **is**

(a) $\frac{28}{81}$ (b) $\frac{28}{243}$

(c) $-\frac{28}{243}$ (d) $-\frac{28}{81}$

33. In the expansion of $(1+x)^n$ **the coefficient of** p^{th} **and** $(p+1)^{th}$ **terms are respectively *p* and *q*. Then** $p+q=$

(a) $n+3$ (b) $n+1$

(c) $n+2$ (d) n

34. The term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ **will be**

(a) 3/2 (b) 5/4

(c) 5/2 (d) None of these

35. The greatest coefficient in the expansion of $(1+x)^{2n+2}$ **is**

(a) $\frac{(2n)!}{(n!)^2}$ (b) $\frac{(2n+2)!}{\{(n+1)!\}^2}$

(c) $\frac{(2n+2)!}{n!(n+1)!}$ (d) $\frac{(2n)!}{n!(n+1)!}$

36. The coefficient of x^n **in expansion of** $(1+x)(1-x)^n$ **is**

(a) $(-1)^{n-1}n$ (b) $(-1)^n(1-n)$

(c) $(-1)^{n-1}(n-1)^2$ (d) $(n-1)$

37. The coefficient of x^4 **in the expansion of** $(1+x+x^2+x^3)^n$ **is**

(a) ${}^nC_4$ (b) ${}^nC_4+{}^nC_2$

(c) ${}^nC_4+{}^nC_2+{}^nC_4.{}^nC_2$ (d) ${}^nC_4+{}^nC_2+{}^nC_1.{}^nC_2$

38. If n is even positive integer, then the condition that the greatest term in the expansion of $(1+x)^n$ may have the greatest coefficient also, is

(a) $\frac{n}{n+2} < x < \frac{n+2}{n}$ (b) $\frac{n+1}{n} < x < \frac{n}{n+1}$

(c) $\frac{n}{n+4} < x < \frac{n+4}{4}$ (d)None of these

39. The term independent of x in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$ is

(a) $C_0^2 + 2C_1^2 + ... + (n+1)C_n^2$ (b) $(C_0 + C_1 + ... + C_n)^2$

(c) $C_0^2 + C_1^2 + ... + C_n^2$ (d) None of these

40. The coefficient of t^{24} in the expansion of $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is

(a) ${}^{12}C_6 + 2$ (b) ${}^{12}C_5$

(c) ${}^{12}C_6$ (d) ${}^{12}C_7$

41. $C_1 + 2C_2 + 3C_3 + 4C_4 + ... + nC_n =$

(a) 2^n (b) $n.\ 2^n$

(c) $n.\ 2^{n-1}$ (d) $n.\ 2^{n+1}$

42. The sum to $(n+1)$ terms of the following series $\frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \frac{C_3}{5} + ...$ is

(a) $\frac{1}{n+1}$ (b) $\frac{1}{n+2}$

(c) $\frac{1}{n(n+1)}$ (d) None of these

43. The value of $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + ...$ is equal to

(a) $\frac{2^n - 1}{n+1}$ (b) $n.2^n$

(c) $\frac{2^n}{n}$ (d) $\frac{2^n + 1}{n+1}$

44. $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ... + C_{n-r}C_n =$

(a) $\frac{(2n)!}{(n-r)!(n+r)!}$ (b) $\frac{n!}{(-r)!(n+r)!}$

(c) $\frac{n!}{(n-r)!}$ (d) None of these

45. ${}^nC_0 - \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 - ...... + (-1)^n \frac{{}^nC_n}{n+1} =$

(a) n (b) $1/n$

(c) $\frac{1}{n+1}$ (d) $\frac{1}{n-1}$

46. If $(1+x)^{15} = C_0 + C_1x + C_2x^2 + ... + C_{15}x^{15}$, then $C_2 + 2C_3 + 3C_4 + ... + 14C_{15} =$

(a) 14.2^{14} (b) $13.2^{14}+1$

(c) $13.2^{14}-1$ (d) None of these

47. If the sum of the coefficients in the expansion of $(\alpha^2x^2 - 2\alpha x + 1)^{51}$ vanishes, then the value of α is

(a) 2 (b) –1

(c) 1 (d) – 2

48. If the sum of the coefficients in the expansion of $(x - 2y + 3z)^n$ is 128 then the greatest coefficient in the expansion of $(1+x)^n$ is

(a) 35 (b) 20

(c) 10 (d) None of these

49. The sum of coefficients in the expansion of $(x + 2y + 3z)^8$ is

(a) 3^8 (b) 5^8

(c) 6^8 (d) None of these

50. The sum of coefficients in the expansion of $(1 + x + x^2)^n$ is

(a) 2 (b) 3^n

(c) 4^n (d) 2^n

51. If $s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n}$ is equal to

(a) $\frac{2n-1}{2}$ (b) $\frac{1}{2}n - 1$

(c) $n-1$ (d) $\frac{1}{2}n$

52. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then the value of $C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n$ will be

(a) $(n+2)2^{n-1}$ (b) $(n+1)2^n$

(c) $(n+1)2^{n-1}$ (d) $(n+2)2^n$

53. **If** $a_k = \frac{1}{k(k+1)}$, **for** $k = 1, 2, 3, 4, \ldots\ldots, n$, **then** $\left(\sum_{k=1}^{n} a_k\right)^2 =$

(a) $\left(\frac{n}{n+1}\right)$ (b) $\left(\frac{n}{n+1}\right)^2$

(c) $\left(\frac{n}{n+1}\right)^4$ (d) $\left(\frac{n}{n+1}\right)^6$

54. **The value of** ${}^{15}C_0^2 - {}^{15}C_1^2 + {}^{15}C_2^2 - \ldots - {}^{15}C_{15}^2$ **is**

(a) 15 (b) – 15 (c) 0 (d) 51

55. **In the expansion of** $(1+x)^5$**, the sum of the coefficient of the terms is**

(a) 80 (b) 16

(c) 32 (d) 64

56. $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1} =$

(a) $\frac{2^n}{n+1}$ (b) $\frac{2^n - 1}{n+1}$

(c) $\frac{2^{n+1} - 1}{n+1}$ (d) None of these

57. **Coefficients of** $x^r [0 \le r \le (n-1)]$ **in the expansion of** $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \ldots + (x+2)^{n-1}$

(a) ${}^nC_r(3^r - 2^n)$ (b) ${}^nC_r(3^{n-r} - 2^{n-r})$

(c) ${}^nC_r(3^r + 2^{n-r})$ (d) None of these

58. **The expansion of** $\frac{1}{(4-3x)^{1/2}}$ **binomial theorem will be valid, if**

(a) $x < 1$ (b) $|x| < 1$

(c) $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$ (d) None of these

59. **In the expansion of** $\left(\frac{1+x}{1-x}\right)^2$**, the coefficient of** x^n **will be**

(a) $4n$ (b) $4n-3$

(c) $4n+1$ (d) None of these

60. $\left(\frac{a}{a+x}\right)^{\frac{1}{2}} + \left(\frac{a}{a-x}\right)^{\frac{1}{2}} =$

(a) $2 + \frac{3x^2}{4a^2} + \ldots$ (b) $1 + \frac{3x^2}{8a^2} + \ldots$

(c) $2 + \frac{x}{a} + \frac{3x^2}{4a^2} + \ldots$ (d) $2 - \frac{x}{a} + \frac{3x^2}{4a^2} + \ldots\ldots$

61. $(r+1)^{th}$ **term in the expansion of** $(1-x)^{-4}$ **will be**

(a) $\frac{x^r}{r!}$ (b) $\frac{(r+1)(r+2)(r+3)}{6}x^r$

(c) $\frac{(r+2)(r+3)}{2}x^r$ (d) None of these

62. $\sum_{k=1}^{n} k\left(1+\frac{1}{n}\right)^{k-1} =$

(a) $n(n-1)$ (b) $n(n+1)$ (c) n^2 (d) $(n+1)^2$

63. If $|x|<1$**, then the value of**

$1+n\left(\frac{2x}{1+x}\right)+\frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2+.....\infty$ **will be**

(a) $\left(\frac{1+x}{1-x}\right)^n$ (b) $\left(\frac{2x}{1+x}\right)^n$

(c) $\left(\frac{1+x}{2x}\right)^n$ (d) $\left(\frac{1-x}{1+x}\right)^n$

64. $1+\frac{1}{3}x+\frac{1.4}{3.6}x^2+\frac{1.4.7}{3.6.9}x^3+....$ **is equal to**

(a) x (b) $(1+x)^{1/3}$

(c) $(1-x)^{1/3}$ (d) $(1-x)^{-1/3}$

65. $1+\frac{1}{4}+\frac{1.3}{4.8}+\frac{1.3.5}{4.8.12}+..........=$

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

66. The coefficient of x^3 **in the expansion of** $\frac{(1+3x)^2}{1-2x}$ **will be**

(a) 8 (b) 32

(c) 50 (d) None of these

67. Coefficient of x^r **in the expansion of** $(1-2x)^{-1/2}$ **is**

(a) $\frac{(2r)!}{(r!)^2}$ (b) $\frac{(2r)!}{2^r(r!)^2}$

(c) $\frac{(2r)!}{(r!)^2 2^{2r}}$ (d) $\frac{(2r)!}{2^r.(r+1)!.(r-1)!}$

68. If $|x|<1$, then in the expansion of $(1+2x+3x^2+4x^3+....)^{1/2}$, the coefficient of x^n is

(a) n (b) $n+1$

(c) 1 (d) – 1

69. $1-\frac{1}{8}+\frac{1}{8}.\frac{3}{16}-\frac{1.3.5}{8.16.24}+.....=$

(a) $\frac{2}{5}$ (b) $\frac{\sqrt{2}}{5}$

(c) $\frac{2}{\sqrt{5}}$ (d) None of these

70. If a_r is the coefficient of x^r, in the expansion of $(1+x+x^2)^n$, then $a_1-2a_2+3a_3-....-2n\,a_{2n}=$

(a) 0 (b) n (c) $-n$ (d) $2n$

71. The number of terms in the expansion of $(a+b+c)^n$ will be

(a) $n+1$ (b) $n+3$

(c) $\frac{(n+1)(n+2)}{2}$ (d) None of these

72. If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, then $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}$ =

(a) $\frac{a_2}{a_2+a_3}$ (b) $\frac{1}{2}\frac{a_2}{(a_2+a_3)}$

(c) $\frac{2a_2}{a_2+a_3}$ (d) $\frac{2a_3}{a_2+a_3}$

73. Let $R=(5\sqrt{5}+11)^{2n+1}$ and $f=R-[R]$, where [.] denotes the greatest integer function. The value of *R.f* is

(a) 4^{2n+1} (b) 4^{2n}

(c) 4^{2n-1} (d) 4^{-2n}

74. If the three consecutive coefficient in the expansion of $(1+x)^n$ are 28, 56 and 70, then the value of *n* is

(a) 6 (b) 4

(c) 8 (d) 10

75. The greatest integer less than or equal to $(\sqrt{2}+1)^6$ is

(a) 196 (b) 197

(c) 198 (d) 199

76. Find the value of

$$\frac{(18^3+7^3+3.18.7.25)}{3^6+6.243.2+15.81.4+20.27.8+15.9.16+6.3.32+64}$$

(a) 1 (b) 5

(c) 25 (d) 100

77. The number of integral terms in the expansion of $\left(\sqrt{3}+\sqrt[8]{5}\right)^{256}$ **is**

(a) 32 (b) 33

(c) 34 (d) 35

78. If $T_0, T_1, T_2, ...T_n$ **represent the terms in the expansion of** $(x+a)^n$**, then** $(T_0-T_2+T_4-....)^2+(T_1-T_3+T_5-....)^2=$

(a) (x^2+a^2) (b) $(x^2+a^2)^n$

(c) $(x^2+a^2)^{1/n}$ (d) $(x^2+a^2)^{-1/n}$

79. The number of non-zero terms in the expansion of $(1+3\sqrt{2}x)^9+(1-3\sqrt{2}x)^9$ **is**

(a) 9 (b) 0

(c) 5 (d) 10

80. $x^5+10x^4a+40x^3a^2+80x^2a^3+80xa^4+32a^5=$

(a) $(x+a)^5$ (b) $(3x+a)^5$

(c) $(x+2a)^5$ (d) $(x+2a)^3$

81. The value of $(\sqrt{5}+1)^5-(\sqrt{5}-1)^5$ **is**

(a) 252 (b) 352

(c) 452 (d) 532

82. $(1+x)^n-nx-1$ **divisible (where** $n\in N$**)**

(a) by $2x$ (b) by x^2

(c) by $2x^3$ (d) All of these

83. The total number of terms in the expansion of $(x+a)^{100}+(x-a)^{100}$ **after simplification will be**

(a) 202 (b) 51

(c) 50 (d) None of these

84. $(\sqrt{2}+1)^6-(\sqrt{2}-1)^6=$

(a) 101 (b) $70\sqrt{2}$

(c) $140\sqrt{2}$ (d) $120\sqrt{2}$

BINOMIAL THEOREM

HINTS AND SOLUTIONS

1.(d). $|T_3| = |T_4|$

2.(a). $T_{r+1} + T_{r+3} = 2T_{r+2}$

3. (b).

4. (c) $T_{16} = {}^{17}C_{15}(\sqrt{x})^2(-\sqrt{y})^{15}$

$= -\frac{17\times16}{2\times1}\times xy^{15/2} = -136\,xy^{15/2}$

5. (d) $T_r = {}^{14}C_{r-1}\,x^{r-1}$; $T_{r+1} = {}^{14}C_r\,x^r$; $T_{r+2} = {}^{14}C_{r+1}\,x^{r+1}$

By the given condition $2.{}^{14}C_r = {}^{14}C_{r-1} + {}^{14}C_{r+1}$ …..(i)

$\Rightarrow 2.\frac{14!}{r!\,(14-r)!} = \frac{14!}{(r-1)!\,(15-r)!} + \frac{14!}{(r+1)!(13-r)!}$

$\Rightarrow 4r^2 - 56r + 180 = 0 \Rightarrow r^2 - 14r + 45 = 0$

$\Rightarrow (r-5)(r-9) = 0 \Rightarrow r = 5, 9$

But 5 is not given. Hence $r = 9$.

6. (a) Coefficient of x^p is ${}^{(p+q)}C_p$ and coefficient of x^q is ${}^{(p+q)}C_q$. But ${}^{(p+q)}C_p = {}^{(p+q)}C_q$, $(\because {}^nC_r = {}^nC_{n-r})$.

7. (c) ${}^{20}C_{r-1} = {}^{20}C_{r+3} \Rightarrow 20 - r + 1 = r + 3 \Rightarrow r = 9$.

8. (d) Ratio of coefficient of $x^{n-r}a^r$ and $x^r a^{n-r}$ is

$= \frac{{}^nC_r}{{}^nC_{n-r}} = \frac{{}^nC_r}{{}^nC_r} = \frac{1}{1}$

9. (a) ${}^{15}C_{2r+2} = {}^{15}C_{r-2}$

But ${}^{15}C_{2r+2} = {}^{15}C_{15-(2r+2)} = {}^{15}C_{13-2r}$

$\Rightarrow {}^{15}C_{13-2r} = {}^{15}C_{r-2} \Rightarrow r = 5$.

10. (a) In the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$, the general term is $T_{r+1} = {}^{10}C_r\left(\frac{x}{2}\right)^{10-r}.\left(-\frac{3}{x^2}\right)^r$

$= {}^{10}C_r(-1)^r.\frac{3^r}{2^{10-r}}x^{10-r-2r}$

Here, the exponent of x is $10 - 3r = 4 \Rightarrow r = 2$

$\therefore\ T_{2+1} = {}^{10}C_2\left(\frac{x}{2}\right)^8\left(-\frac{3}{x^2}\right)^2 = \frac{10.9}{1.2}.\frac{1}{2^8}.3^2.x^4$

$= \frac{405}{256}x^4$

$\therefore$ The required coefficient $= \frac{405}{256}$.

11. (b) For number of term,

$(11-r)(1)+r(-2) = -7 \Rightarrow 11-r-2r = -7 \Rightarrow r = 6$

Thus coefficient of x^{-7} is ${}^{11}C_6(a)^5\left(-\frac{1}{b}\right)^6 = \frac{462}{b^6}a^5$

12. (b) $\frac{\text{Coefficient of } x^n \text{ in expansion of} (1+x)^{2n}}{\text{Coefficient of } x^n \text{ in expansion of} (1+x)^{2n-1}}$

$= \frac{{}^{2n}C_n}{{}^{(2n-1)}C_n} = \frac{(2n)!}{n!n!} \times \frac{(n-1)!n!}{(2n-1)!}$

$= \frac{(2n)(2n-1)!(n-1)!}{n(n-1)!\,(2n-1)!} = \frac{2n}{n} = 2:1$

$\Rightarrow \frac{A}{B} = \frac{2}{1} \Rightarrow A = 2B$.

13. (b) Coefficient of $p^{th}, (p+1)^{th}$ and $(p+2)^{th}$ terms in expansion of $(1+x)^n$ are ${}^nC_{p-1}, {}^nC_p, {}^nC_{p+1}$.

Then $2\,{}^nC_p = {}^nC_{p-1} + {}^nC_{p+1}$

$\Rightarrow n^2 - n(4p+1) + 4p^2 - 2 = 0$

14. (c) Coefficient of $T_5 = {}^nC_4, T_6 = {}^nC_5$ and $T_7 = {}^nC_6$

According to the condition, $2\ {}^nC_5 = {}^nC_4 + {}^nC_6$

After solving, we get n=7 or 14.

15. (c) $T_4 = T_{3+1} = {}^nC_3\ a^{n-3}b^3$

$\Rightarrow {}^nC_3 = 56 \Rightarrow \frac{n!}{3!\ (n-3)!} = 56$

$\Rightarrow n(n-1)(n-2) = 56.6 \Rightarrow n(n-1)(n-2) = 8.7.6$

$\Rightarrow n = 8$.

16. (b) We have $(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + ...$

By hypothesis, $\frac{m(m-1)}{2}x^2 = -\frac{1}{8}x^2$

$\Rightarrow 4m^2 - 4m = -1 \Rightarrow (2m-1)^2 = 0 \Rightarrow m = \frac{1}{2}$.

17. (c) $T_r = {}^{15}C_{r-1}(x^4)^{16-r}\left(\frac{1}{x^3}\right)^{r-1} = {}^{15}C_{r-1}x^{67-7r}$

$\Rightarrow 67 - 7r = 4 \Rightarrow r = 9$.

18. (a) Coefficient of $(2r+1)^{th}$ term in expansion of $(1+x)^{43} = {}^{43}C_{2r}$ and coefficient of $(r+2)^{th}$ term = coefficient of $\{(r+1)+1\}^{th}$ term = ${}^{43}C_{r+1}$

According to question ${}^{43}C_{2r} = {}^{43}C_{r+1} = {}^{43}C_{43-(r+1)}$

Then $2r = 43-(r+1)$ or $3r = 42$ or $r = 14$.

19. (c) $T_{r+1} = {}^{2n}C_r x^{2n-r}\left(\frac{1}{x^2}\right)^r = {}^{2n}C_r x^{2n-3r}$,

This contains x^m, if $2n - 3r = m$

i.e. if $r = \frac{2n-m}{3}$

$\therefore$ Coefficient of $x^m = {}^{2n}C_r,\ r = \frac{2n-m}{3}$

$= \frac{2n!}{(2n-r)!r!} = \frac{2n!}{\left(2n - \frac{2n-m}{3}\right)!\left(\frac{2n-m}{3}\right)!}$

$= \frac{2n!}{\left(\frac{4n+m}{3}\right)!\left(\frac{2n-m}{3}\right)!}$.

20. (b) x^7, x^8 will occur in T_8 and T_9.

Coefficients of T_8 and T_9 are equal.

$\therefore {}^nC_7 2^{n-7}\left(\frac{1}{3}\right)^7 = {}^nC_8 2^{n-8}\left(\frac{1}{3}\right)^8 \Rightarrow n = 55$.

21. (b) In the expansion of $\left(x^2 + \frac{a}{x}\right)^5$ the general term is $T_{r+1} = {}^5C_r(x^2)^{5-r}\left(\frac{a}{x}\right)^r = {}^5C_r a^r x^{10-3r}$

Here, exponent of x is $10 - 3r = 1 \Rightarrow r = 3$

$\therefore T_{2+1} = {}^5C_3 a^3 x = 10a^3.x$

Hence coefficient of $x$is $10a^3$.

22. (b) $T_2 = {}^{2n}C_1\ x$, $T_3 = {}^{2n}C_2\ x^2$, $T_4 = {}^{2n}C_3\ x^3$

Coefficient of T_2, T_3,$T_4$are in A.P.

$\Rightarrow 2.{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$\Rightarrow 2\frac{2n!}{2!(2n-2)!} = \frac{2n!}{(2n-1)!} + \frac{2n!}{3!(2n-3)!}$

$\Rightarrow 2n^2 - 9n + 7 = 0$.

23. (c) $(1+x)^m(1-x)^n$

$$= \left(1 + mx + \frac{m(m-1)x^2}{2!} + ...\right)\left(1 - nx + \frac{n(n-1)}{2!}x^2 - ....\right)$$

$$= 1 + (m-n)x + \left[\frac{n^2 - n}{2} - mn + \frac{(m^2 - m)}{2}\right]x^2 + ...$$

Given, $m - n = 3$ or $n = m - 3$

Hence $\frac{n^2 - n}{2} - mn + \frac{m^2 - m}{2} = -6$

$\Rightarrow \frac{(m-3)(m-4)}{2} - m(m-3) + \frac{m^2 - m}{2} = -6$

$\Rightarrow m^2 - 7m + 12 - 2m^2 + 6m + m^2 - m + 12 = 0$

$\Rightarrow -2m + 24 = 0 \Rightarrow m = 12$

24. (a)

25. (a)

26. (a)

27. (c) $(1+x)^{21} + (1+x)^{22} + ... + (1+x)^{30}$

$$= (1+x)^{21}\left[\frac{(1+x)^{10} - 1}{(1+x) - 1}\right] = \frac{1}{x}[(1+x)^{31} - (1+x)^{21}]$$

$\therefore$ Coefficient of x^5 in the given expression

= Coefficient of x^5 in $\left\{\frac{1}{x}[(1+x)^{31} - (1+x)^{21}]\right\}$

= Coefficient of x^6 in $[(1+x)^{31} - (1+x)^{21}]$

$= {}^{31}C_6 - {}^{21}C_6$.

28. (d) $(1 + 3x + 2x^2)^6 = [1 + x(3 + 2x)]^6$

Only x^{11} gets from ${}^6C_6 x^6 (3 + 2x)^6$

$\because {}^6C_6 x^6 (3+2x)^6 = x^6 (3+2x)^6$

$\therefore$ Coefficient of $x^{11} = {}^6C_5 3.2^5 = 576$.

29. (b) Middle term of $\left(x + \frac{1}{x}\right)^{10}$ is $T_6 = {}^{10}C_5$.

30. (d) $T_2 = n(x)^{n-1}(a)^1 = 240$ .....(i)

$T_3 = \frac{n(n-1)}{1.2} x^{n-2} a^2 = 720$ .....(ii)

$T_4 = \frac{n(n-1)(n-2)}{1.2.3} x^{n-3} a^3 = 1080$ .....(iii)

To eliminate x,

$\frac{T_2 . T_4}{T_3^2} = \frac{240 . 1080}{720 . 720} = \frac{1}{2} \Rightarrow \frac{T_2}{T_3} . \frac{T_4}{T_3} = \frac{1}{2}$

Now, $\frac{T_{r+1}}{T_r} = \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

Putting $r = 3$ and 2 in above expression, we get

$\Rightarrow \frac{n-2}{3} . \frac{2}{n-1} = \frac{1}{2} \Rightarrow n = 5$.

31. (d) We have $(x)^{12-r}\left(\frac{1}{x^2}\right)^r = x^0 \Rightarrow x^{12-3r} = x^0 \Rightarrow r = 4$

Hence the required term is ${}^{12}C_4 2^8 \left(-\frac{1}{2}\right)^4 = 7920$.

32. (b) In $\left(x^2 - \frac{1}{3x}\right)^9$,

$T_{r+1} = {}^9C_r (x^2)^{9-r} \left(-\frac{1}{3x}\right)^r = {}^9C_r x^{18-2r} \frac{(-1)^r}{3^r} x^{-r}$

It is independent of x.

$\therefore 18 - 3r = 0 \Rightarrow r = 6$

$\therefore T_7 = {}^9C_6 x^{18-12} \frac{(-1)^6}{3^6} x^{-6} = {}^9C_6 \frac{(-1)^6}{36} = \frac{28}{243}$

33. (b) p^{th} term $= T_p = {}^nC_{p-1} (x)^{n-p+1} (1)^{p-1} = p$

$(p + 1)^{th}$ term $= T_{p+1} = {}^nC_p (x)^{n-p} (1)^p = q$

Then, coefficient of $\frac{p}{q} = \frac{{}^nC_{p-1}}{{}^nC_p}$

$\Rightarrow \frac{p}{q} = \frac{n!}{(p-1)!(n-p+1)!} . \frac{p!\ (n-p)!}{n!}$

$\Rightarrow \frac{p}{q} = \frac{p}{n-p+1}$

$\Rightarrow p + q = n + 1$.

34. (b) $(10 - r)\left(\frac{1}{2}\right) + r(-2) = 0 \Rightarrow 5 - \frac{r}{2} - 2r = 0 \Rightarrow r = 2$

So the term independent of x

$={}^{10}C_2\times\left(\frac{1}{3}\right)^4\left(\frac{3}{2}\right)^2=\frac{10\times 9}{2\times 1}\times\frac{1}{3\times 3\times 2\times 2}=\frac{5}{4}$

35. (b) Greatest coefficient of $(1+x)^{2n+2}$ is

$={}^{(2n+2)}C_{n+1}=\frac{(2n+2)!}{\{(n+1)!\}^2}$

36. (b) Coefficient of x^n in expansion of $(1+x)(1-x)^n$

i.e., coefficient of x^n in expansion of $(1-x)^n$ + coefficient of x^{n-1} in expansion of $(1-x)^n$

Now, $(-1)^n\, {}^nC_n+(-1)^{n-1}\ {}^nC_{n-1}$

$(-1)^n[{}^nC_n-{}^nC_{n-1}]=(-1)^n[1-n]$.

37. (d) $(1+x+x^2+x^3)^n=\{(1+x)^n(1+x^2)^n\}$

$=(1+{}^nC_1x+{}^nC_2x^2+....+{}^nC_nx^n)$

$(1+{}^nC_1x^2+{}^nC_2x^4+....+{}^nC_nx^{2n})$

Therefore the coefficient of x^4

$={}^nC_2+{}^nC_2.{}^nC_1+{}^nC_4={}^nC_4+{}^nC_2+{}^nC_1.{}^nC_2$

38. (a) If n is even, the greatest coefficient is ${}^nC_{n/2}$

Therefore the greatest term $={}^nC_{n/2}x^{n/2}$

$\therefore\ {}^nC_{n/2}x^{n/2}>{}^nC_{(n/2)-1}x^{(n-2)/2}$

and ${}^nC_{n/2}x^{n/2}>{}^nC_{(n/2)+1}x^{(n/2)+1}$

$\Rightarrow\frac{n-\frac{n}{2}+1}{\frac{n}{2}}x>1$ and $\frac{\frac{n}{2}}{\frac{n}{2}+1}x<1$

$\Rightarrow x>\frac{\frac{n}{2}}{\frac{n}{2}+1}$ and $x<\frac{\frac{n}{2}+1}{\frac{n}{2}}$

$\Rightarrow x>\frac{n}{n+2}$ and $x<\frac{n+2}{n}$

39. (c) As in Previous question, obviously the term independent of x will be

${}^nC_0.{}^nC_0+{}^nC_1.{}^nC_1+.....{}^nC_n.{}^nC_n=C_0^2+C_1^2+....+C_n^2$.

40. (a) $(1+t^2)^{12}(1+t^{12})(1+t^{24})$

$=(1+{}^{12}C_1t^2+{}^{12}C_2t^4+...+{}^{12}C_4t^8+...+{}^{12}C_{10}t^{20}+....)\ (1+t^{12}+t^{24}+t^{36})$

$\therefore$ Coefficient of $t^{24}={}^{12}C_6+2$.

41. (c) Put $n = 1, 2, 3, ...$

$S_1 = 1, S_2 = 2 + 2 = 4$

Now by alternate (c), put $n = 1, 2$

$S_1 = 1.2^0 = 1, S_2 = 2.2^1 = 4$

42. (d) $(1-x)^n = C_0 - C_1 x + C_2 x^2 - C_3 x^3 + ...$

$\Rightarrow x\,(1-x)^n = C_0 x - C_1 x^2 + C_2 x^3 - C_3 x^4 + ...$

$\Rightarrow \int_0^1 x(1-x)^n\,dx = \int_0^1 (C_0 x - C_1 x^2 + C_2 x^3 ....)dx$ ......(i)

The integral on the LHS

$= \int_1^0 (1-t)t^n(-dt)$, by putting $1 - x = t$

$= \int_0^1 (t^n - t^{n+1})dt = \frac{1}{n+1} - \frac{1}{n+2}$

Whereas the integral on the RHS of (i)

$= \left[\frac{C_0 x^2}{2} - \frac{C_1 x^3}{3} + \frac{C_2 x^4}{4} - ....\right] = \frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - ....$

$\therefore \frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - ....$ to $(n+1)$ terms

$= \frac{1}{n+1} - \frac{1}{n+2} = \frac{1}{(n+1)(n+2)}$.

43. (a) We know that

$\frac{(1+x)^n - (1-x)^n}{2} = C_1 x + C_3 x^3 + C_5 x^5 + ...$

Integrating from $x = 0$ to $x = 1$, we get

$\frac{1}{2}\int_0^1 \{(1+x)^n - (1-x)^n\}\,dx$

$= \int_0^1 (C_1 x + C_3 x^3 + C_5 x^5 + ...)dx$

$\Rightarrow \frac{1}{2}\left\{\frac{(1+x)^{n+1}}{n+1} + \frac{(1-x)^{n+1}}{n+1}\right\}_0^1 = \frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + ...$

or $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + ... = \frac{1}{2}\left\{\frac{2^{n+1}-1}{n+1} + \frac{0-1}{n+1}\right\}$

$= \frac{1}{2}\left(\frac{2^{n+1}-2}{n+1}\right) = \frac{2^n - 1}{n+1}$

44. (a) $(1+x)^n = C_0 + C_1 x + C_2 x^2 + ... + C_r x^r + ...$ .....(i)

$$\left(1+\frac{1}{x}\right)^n = C_0 + C_1\frac{1}{x} + C_2\frac{1}{x^2} + ... + C_r\frac{1}{x^r} + ...$$ .....(ii)

Multiplying both sides and equating coefficient of x^r in $\frac{1}{x^n}(1+x)^{2n}$ or the coefficient of x^{n+r} in $(1+x)^{2n}$ we get the value of required expression

$= {}^{2n}C_{n+r} = \frac{(2n)!}{(n-r)!(n+r)!}$

Trick: Solving conversely.

Put $n=1$ and $r=0$ in first term, (given condition)

(i) ${}^1C_0{}^1C_0 + {}^1C_1{}^1C_1 = 1+1=2$, $(\because r \le n)$

Put $n=2, r=1$, then

(ii) ${}^2C_0{}^2C_1 + {}^2C_1{}^2C_2 = 2+2=4$

Now check the options

(a) (i) Put $n=1, r=0$, we get $\frac{2!}{(1)!\,(1)!} = 2$

(ii) Put $n=2, r=1$, we get $\frac{4!}{(1)!\,(3)!} = 4$

45. (c) Trick : Put $n=$ 1, 2

At $n=1, {}^1C_0 - \frac{1}{2}{}^1C_1 = 1-\frac{1}{2} = \frac{1}{2}$

At $n=2,\ {}^2C_0 - \frac{1}{2}\,{}^2C_1 + \frac{1}{3}\,{}^2C_2 = 1-1+\frac{1}{3} = \frac{1}{3}$

Which is given by option (c).

46. (b) We have $(1+x)^{15} = C_0 + C_1 x + C_2 x^2 . + ... + C_{15} x^{15}$

$\Rightarrow \frac{(1+x)^{15}-1}{x} = C_1 + C_2 x + ... + C_{15} x^{14}$

Differentiating both sides with respect to *x*, we get

$= \frac{x.15(1+x)^{14} - (1+x)^{15} + 1}{x^2}$

$= C_2 + 2C_3 x + ... + {}^{14}C_{15} x^{13}$

Putting $x=1$, we get

$C_2 + 2C_3 + ... + 14C_{15} = 15.2^{14} - 2^{15} + 1 = 13.2^{14} + 1.$

47. (c) The sum of the coefficients of the polynomial $(\alpha^2 x^2 - 2\alpha x + 1)^{51}$ is obtained by putting $x=1$ in $(\alpha^2 x^2 - 2\alpha x + 1)^{51}$.

Therefore by hypothesis $(\alpha^2-2\alpha+1)^{51}=0 \Rightarrow \alpha=1$

48. (a) Sum of the coefficient in the expansion

$(x-2y+3z)^n$ is $(1-2+3)^2=2^n$

$\therefore\ 2^n=128 \Rightarrow n=7$

Therefore, greatest coefficient in the expansion of $(1+x)^7$ is ${}^7C_3$ or ${}^7C_4$ because both are equal to 35.

49. (c) Sum of the coefficients is obtained by putting $x=y=z=1$, so sum of the coefficients $=(1+2+3)^8=6^8$.

50. (b) We can obtain sum of coefficients by putting x = 1 in polynomial.

$\Rightarrow (1+x+x^2)^n=(1+1+1)^n=3^n$.

51. (d) We have, $S_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}$

$$t_n=\sum_{r=0}^{n}\frac{n-(n-r)}{{}^nC_{n-r}}, \qquad [\because {}^nC_r={}^nC_{n-r}]$$

$$=n\sum_{r=0}^{n}\frac{1}{{}^nC_r}-\sum_{r=0}^{n}\frac{n-r}{{}^nC_{n-r}}$$

$$t_n=n.S_n-\left[\frac{n}{{}^nC_n}+\frac{n-1}{{}^nC_{n-1}}+.....+\frac{1}{{}^nC_1}+0\right]$$

$$t_n=n.S_n-\sum_{r=0}^{n}\frac{r}{{}^nC_r}$$

$$\Rightarrow t_n=n.S_n-t_n \Rightarrow 2t_n={}^nS_n \Rightarrow \frac{t_n}{S_n}=\frac{n}{2}.$$

52. (a) Trick: Put n=1, the expression is equivalent to ${}^1C_0+2.{}^1C_1=1+2=3$

Only option (a) gives the value.

53. (b) $\sum_{k=1}^{n}a_k=\sum_{k=1}^{n}\frac{1}{k\,(k+1)}$

$$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+...+\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=1-\frac{1}{n+1}=\frac{n}{n+1}$$

$$\left(\sum_{k=1}^{n}a_k\right)^2=\left(\frac{n}{n+1}\right)^2.$$

54. (c) As we know that

$^nC_0 - {}^nC_1^2 + {}^nC_2^2 - {}^nC_3^2 + ... + (-1)^n . {}^nC_n^2 = 0$, (if n is odd)

And in the question n=15 (odd).

55. (c) Sum of the coefficients = $(1+1)^5 = 2^5 = 32$.

56. (c) Proceeding as above and putting $n+1=N$.

So given term can be written as

$\frac{1}{N}\{{}^NC_1 + {}^NC_2 + {}^NC_3 + ...\}$

57. (b) We have $(x+3)^{n-1} + (x+3)^{n-2}(x+2) +$

$(x+3)^{n-3}(x+2)^2 + ... + (x+2)^{n-1}$

$= \frac{(x+3)^n - (x+2)^n}{(x+3)-(x+2)} = (x+3)^n - (x+2)^n$

$(\because \frac{x^n - a^n}{x-a} = x^{n-1} + x^{n-2}a^1 + x^{n-3}a^2 + ... + a^{n-1})$

Therefore coefficient of x^r in the given expression

= Coefficient of x^r in $[(x+3)^n - (x+2)^n]$

$= {}^nC_r 3^{n-r} - {}^nC_r 2^{n-r} = {}^nC_r(3^{n-r} - 2^{n-r})$

58. (d) The given expression can be written as $4^{-1/2}\left(1-\frac{3}{4}x\right)^{-1/2}$ and it is valid only when $\left|\frac{3}{4}x\right| < 1 \Rightarrow -\frac{4}{3} < x < \frac{4}{3}$.

59. (a) Given term can be written as $(1+x)^2(1-x)^{-2}$

$= (1+2x+x^2)[1+2x+3x^2+....+(n-1)x^{n-2}$

$= x^n(n+1+2n+n-1)+....$

Therefore coefficient of x^n is $4n$.

60. (a) $\left(\frac{a+x}{a}\right)^{-1/2} + \left(\frac{a-x}{a}\right)^{-1/2} = \left(1+\frac{x}{a}\right)^{-1/2} + \left(1-\frac{x}{a}\right)^{-1/2}$

$$= \left[1 + \left(-\frac{1}{2}\right)\left(\frac{x}{a}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2.1}\left(\frac{x}{a}\right)^2 + ...\right] + \left[1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{a}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2.1}\left(-\frac{x}{a}\right)^2 + ...\right]$$

$= 2 + \frac{3x^2}{4a^2} + ...$

Here odd terms cancel each other.

61. (b) $(1-x)^{-4}=\left[\frac{1.2.3}{6}x^0+\frac{2.3.4}{6}x+\frac{3.4.5}{6}x^2+\frac{4.5.6}{6}x^3+...+\frac{(r+1)(r+2)(r+3)}{6}x^r+....\right]$

Therefore $T_{r+1}=\frac{(r+1)(r+2)(r+3)}{6}x^r$.

62. (c) $\sum_{k=1}^{n}k\left(1+\frac{1}{n}\right)^{k-1}$

$=1+2\left(1+\frac{1}{n}\right)^1+3\left(1+\frac{1}{n}\right)^2+....$ up to n terms

= $1+2t+3t^2+...$ up to n terms

= $(1-t)^{-2}=\left[1-\left(1+\frac{1}{n}\right)\right]^{-2}$ = $\left(\frac{1}{n}\right)^{-2}=(n)^2=n^2$.

63. (a) $(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+....+{}^nC_nx^n+....$

If x is replaced by $-\left(\frac{2x}{1+x}\right)$ and n is $-n$.

64. (d) Let $(1+y)^n=1+\frac{1}{3}x+\frac{1.4}{3.6}x^2+\frac{1.4.7}{3.6.9}x^3+....$

$=1+ny+\frac{n(n-1)}{2!}y^2+.....$

Comparing the terms, we get

$ny=\frac{1}{3}x,\frac{n(n-1)}{2!}y^2=\frac{1.4}{3.6}x^2$

Solving, $n=-\frac{1}{3},y=-x$.

Hence given series $=(1-x)^{-1/3}$

65. (a) Let the given series be identical with the expansion of $(1+x)^n$ *i.e.* with $1+nx+\frac{n(n-1)}{2!}x^2+....;|x|<1$. Then, $nx=\frac{1}{4}$ and $\frac{n(n-1)}{2}x^2=\frac{1}{4}.\frac{3}{8}=\frac{3}{32}$ Solving these two equations for n and x. We get $x=-\frac{1}{2}$ and $n=-\frac{1}{2}$. $\therefore$ Sum of the given series

= $(1+x)^n=\left(1-\frac{1}{2}\right)^{-1/2}=2^{1/2}=\sqrt{2}.$

66. (c) $=(1+3x)^2(1-2x)^{-1}$

$=(1+3x)^2\left(1+2x+\frac{1.2}{2.1}(-2x)^2+....\right)$

$=(1+6x+9x^2)(1+2x+4x^2+8x^3+....)$

Therefore coefficient of x^3 is $(8+24+18)=50$.

67. (b) Coefficient of $x^r = \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)...\left(-\frac{1}{2}-r+1\right)}{r!}(-2)^r$

$$\frac{1.3.5...(2r-1)}{2^r}.\frac{(-1)^r(-1)^r 2^r}{r!}$$

$$= \frac{1.3.5...(2r-1)}{r!} = \frac{2r!}{r!\ r!\ 2^r}$$

68. .(c) Since $1+2x+3x^2+4x^3+....\infty=(1-x)^{-2}$

Therefore, we have

$$(1+2x+3x^2+4x^3+....\infty)^{1/2} = \{(1-x)^{-2}\}^{1/2}$$

$$=(1-x)^{-1} = 1+x+x^2+....+x^n+....\infty$$

$\therefore$ The coefficient of $x^n = 1$.

69. (c) Comparing the given expression to

$1+nx+\dfrac{n(n-1)}{2!}x^2+....i.e.(1+x)^n$, we get

$nx=-\dfrac{1}{8}$ and $\dfrac{n(n-1)}{2!}x^2=\dfrac{3}{128}\Rightarrow x=\dfrac{1}{4}, n=-\dfrac{1}{2}$

Hence $1-\dfrac{1}{8}+\dfrac{1}{8}.\dfrac{3}{16}-....=\left(1+\dfrac{1}{4}\right)^{-1/2}=\dfrac{2}{\sqrt{5}}$

70. (c) Let us take $a_0+a_1x+a_2x^2+.....+a_{2n}x^{2n}=(1+x+x^2)^n$

Differentiating with respect to x on both sides $a_1+2a_2x+...+2n\,a_{2n}x^{2n-1} = n(1+x+x^2)^{n-1}(2x+1)$

Put $x = -1 \Rightarrow a_1-2a_2+3a_3-....+2n\,a_{2n}=-n$.

71. (c) By converse rule, put n= 1, 2, then number of terms are 3, 6.

Hence number of terms of $(a+b+c)^n = \dfrac{(n+1)(n+2)}{2}$.

72. (c) Let a_1,a_2,a_3,a_4 be respectively the coefficients of $(r+1)^{th},(r+2)^{th}$, $(r+3)^{th}$ and $(r+4)^{th}$ terms in the expansion of $(1+x)^n$.

Then $a_1={}^nC_r, a_2={}^nC_{r+1}, a_3={}^nC_{r+2}, a_4={}^nC_{r+3}$

Now $\dfrac{a_1}{a_1+a_2}+\dfrac{a_3}{a_3+a_4}=\dfrac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}}+\dfrac{{}^nC_{r+2}}{{}^nC_{r+2}+{}^nC_{r+3}}$

$$=\frac{{}^nC_r}{{}^{n+1}C_{r+1}}+\frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}}=\frac{{}^nC_r}{\frac{n+1}{r+1}{}^nC_r}+\frac{{}^nC_{r+2}}{\frac{n+1}{r+3}{}^nC_{r+2}}\quad (\because {}^nC_r=\frac{n}{r}{}^{n-1}C_{r-1})$$

$= \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2(r+2)}{n+1}$

$= 2\frac{{}^nC_{r+1}}{{}^{n+1}C_{r+2}} = 2\frac{{}^nC_{r+1}}{{}^nC_{r+1} + {}^nC_{r+2}} = \frac{2a_2}{a_2 + a_3}$

73. (a) Since $(5\sqrt{5} - 11)(5\sqrt{5} + 11) = 4$

$\Rightarrow 5\sqrt{5} - 11 = \frac{4}{5\sqrt{5} + 11}$,

$\because\ 0 < 5\sqrt{5} - 11 < 1 \Rightarrow 0 < (5\sqrt{5} - 11)^{2n+1} < 1$,

74. (c) Let the three consecutive coefficients be

${}^nC_{r-1} = 28, {}^nC_r = 56$ and ${}^nC_{r+1} = 70$, so that

$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} = \frac{56}{28} = 2$

And $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1} = \frac{70}{56} = \frac{5}{4}$

This gives $n + 1 = 3r$ and $4n - 5 = 9r$

$\therefore \frac{4n-5}{n+1} = 3 \Rightarrow n = 8$

75. (b) Let $(\sqrt{2} + 1)^6 = k + f$, where k is integral part and f the fraction $(0 \le f < 1)$.

Let $(\sqrt{2} - 1)^6 = f', (0 < f' < 1)$,

Since $0 < (\sqrt{2} - 1) < 1$

76. (a) The numerator is of the form

$a^3 + b^3 + 3ab(a+b) = (a+b)^3$

$\therefore N^r = (18 + 7)^3 = 25^3$

$\therefore D^r = 3^6 + {}^6C_1 3^5.2^1 + {}^6C_2 3^4.2^2 \quad + {}^6C_3 3^3.2^3 + {}^6C_4 3^2.2^4 + {}^6C_5 3.2^5 + {}^6C_6 2^6$

This is clearly the expansion of $(3+2)^6 = 5^6 = (25)^3$

$\therefore \frac{N^r}{D^r} = \frac{(25)^3}{(25)^3} = 1$

77. (b) $T_{r+1} = {}^{256}C_r (\sqrt{3})^{256-r} (\sqrt[8]{5})^r = {}^{256}C_r (3)^{\frac{256-r}{2}} (5)^{r/8}$

Terms would be integral if $\frac{256-r}{2}$ and $\frac{r}{8}$ both are positive integer.

As $0 \le r \le 256$, $\therefore r = 0, 8, 16, 24, ..., 256$ For above values of r, $\left(\frac{256-r}{2}\right)$ is also an integer.

$\therefore$ Total number of values of r = 33.

78. (b) From the given condition, replacing a by ai and – ai respectively, we get

$(x + ai)^n = (T_0 - T_2 + T_4 - ....) + i(T_1 - T_3 + T_5 - ....)$ ....(i)

And $(x-ai)^n = (T_0 - T_2 + T_4 - ....) - i(T_1 - T_3 + T_5 - ....)$ ......(ii)

Multiplying (ii) and (i) we get required result

i.e. $(x^2+a^2)^n = (T_0 - T_2 + T_4 - ....)^2 + (T_1 - T_3 + T_5 - ....)^2$

79. (c) Given expression

$= 2[1 + {}^9C_2(3\sqrt{2}x)^2 + {}^9C_4(3\sqrt{2}x)^4 + {}^9C_6(3\sqrt{2}x)^6 + {}^9C_8(3\sqrt{2}x)^8]$

$\therefore$ The number of non-zero terms is 5.

80. (c) Conversely,

$(x+a)^n = {}^nC_0 + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + ...$

So,

$(x+2a)^5 = x^5 + 10x^4a + 40x^3a^2 + 80x^2a^3 + 80xa^4 + 32a^5$.

81. (b) $(\sqrt{5}+1)^5 - (\sqrt{5}-1)^5$

$= 2\left\{{}^5C_1(\sqrt{5})^4 + {}^5C_3(\sqrt{5})^2 + {}^5C_5.1\right\} = 352$

82. (b) $(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \frac{n(n-1)(n-2)}{1.2.3}x^3 + ...$

$\therefore (1+x)^n - nx - 1 = x^2\left[\frac{n(n-1)}{1.2} + \frac{n(n-1)(n-2)}{1.2.3}x + ...\right]$

From above it is clear that $(1+x)^n - nx - 1$ is divisible by x^2.

83. (b) We know $\frac{1}{2}\left\{(1+a)^n + (1-a)^n\right\} = {}^nC_0 + {}^nC_2a^2 + {}^nC_4a^4 + ...$

Therefore, number of terms in expansion of $\{(x+a)^{100} + (x-a)^{100}\}$ is 51.

84. (c) $(x+a)^n - (x-a)^n$

$= 2[{}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a^3 + {}^nC_5x^{n-5}a^5 + ...]$

$\therefore (\sqrt{2}+1)^6 - (\sqrt{2}-1)^6$

$= 2[{}^6C_1(\sqrt{2})^5(1)^1 + {}^6C_3(\sqrt{2})^3(1)^3 + {}^6C_5(\sqrt{2})^1(1)^5]$

$\because (\sqrt{2}+1)^6 - (\sqrt{2}-1)^6 = 2[6\times 4\sqrt{2} + 20\times 2\sqrt{2} + 6\sqrt{2}]$

$= 2[24\sqrt{2} + 40\sqrt{2} + 6\sqrt{2}] = 140\sqrt{2}$. $= \frac{1}{N}\left\{2^N - 1\right\} = \frac{1}{n+1}(2^{n+1} - 1)$ $(\because N = n+1)$